Remote unambiguous discrimination of linearly independent symmetric *d*-level quantum states

2009 J. Phys. A: Math. Theor. 42 055308

(http://iopscience.iop.org/1751-8121/42/5/055308)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 055308 (8pp) doi:10.1088/1751-8113/42/5/055308

Remote unambiguous discrimination of linearly independent symmetric *d*-level quantum states

Libing Chen[1], **Yuhua Liu, Peng Tan and Hong Lu**

Department of Photoelectron and Physics, Foshan University, Foshan 528000, People's Republic of China

E-mail: chlibing2008@163.com

Received 13 June 2008, in final form 24 October 2008
Published 9 January 2009
Online at stacks.iop.org/JPhysA/42/055308

Abstract
A set of linearly independent nonorthogonal symmetric *d*-level quantum states can be discriminated remotely and unambiguously with the aid of two-level Einstein–Podolsky–Rosen (EPR) states. We present a scheme for such a kind of remote unambiguous quantum state discrimination (UD). The probability of discrimination is in agreement with the optimal probability for local unambiguous discrimination among *d* symmetric states (Chefles and Barnett 1998 *Phys. Lett.* A **250** 223). This scheme consists of a remote generalized measurement described by a positive operator valued measurement (POVM). This remote POVM can be realized by performing a nonlocal $2d \times 2d$ unitary operation on two spatially separated systems, one is the qudit which is encoded by one of the *d* symmetric nonorthogonal states to be distinguished and the other is an ancillary qubit, and a conventional local von Neumann orthogonal measurement on the ancilla. By decomposing the evolution process from the initial state to the final state, we construct a quantum network for realizing the remote POVM with a set of two-level nonlocal controlled-rotation gates, and thus provide a feasible physical means to realize the remote UD. A two-level nonlocal controlled-rotation gate can be implemented by using a two-level EPR pair in addition to local operations and classical communications (LOCCs).

PACS numbers: 03.67.Hk, 03.65.Ta

1. Introduction

Quantum state discrimination (QSD) is one of the fundamentally important problems in quantum information science and poses fundamental limitations on the amount of information that can be obtained about the state of a single system. Many novel schemes in quantum

[1] Author to whom any correspondence should be addressed.

communication and computation, such as quantum cryptography [1, 2], quantum teleportation [3] and entanglement concentration [4], etc, are based on the fact that nonorthogonal states cannot be discriminated in a determinate way. A great deal of attention has been attracted into this field in recent years, especially the 'unambiguous quantum state discrimination' (UD). UD is a sort of discrimination that never gives an erroneous result, but sometimes it may fail. UD was pioneered two decades ago by Ivanovic–Dieks–Peres (IDP) for finding the optimal probability of conclusive discrimination between two nonorthogonal states with equal *a priori* probability [5–7]. Jaeger and Shimony have generalized this result by considering the case of states with different *a priori* probability [8]. Chefles and Barnett [9] have generalized IDP's solution to an arbitrary number of equally probable states which are related by a symmetry transformation. The physical methods proposed to do UD include linear optics [10], ion trap architecture [11] and nuclear magnetic resonance [12]. Experiments for discriminating between nonorthogonal polarization states at IDP limit were accomplished utilizing linear optics only [13, 14]. In the same way, the experimental setup of unambiguous discrimination among three nonorthogonal quantum states was carried out by Mohsenit *et al* [15], with a success rate of 55%.

To unambiguously discriminate nonorthogonal quantum states, we need to use general positive operator valued measurement (POVM) instead of orthogonal projectors. A POVM is given by a set of Kraus operators $\{M_i\}$ [16]. For a POVM with n operators, they must satisfy the completeness relation $\sum_{i=1}^{n} M_i^{+} M_i = I$. Each M_i corresponds to a distinct outcome of the operation. A POVM performed on a quantum system B can always be realized by entangling system B with ancillary system A via unitary evolution U_{BA} [16], which usually is a conditional evolution. Postselection (projective measurement) of the ancilla induces an effective nonunitary transformation ε of original system $|\phi\rangle_B$, i.e.

$$\varepsilon[|\phi\rangle_B] = \mathrm{Tr}_A\big[U_{AB}|\phi\rangle_B \otimes |\phi\rangle_A U_{BA}^{+}\big], \tag{1}$$

where $|\phi\rangle_A$ is an appropriately chosen state of ancillary system A and Tr_A denotes the partial trace over ancilla A. By the appropriate design of the entangling unitary, this effective nonunitary transformation can turn an initially nonorthogonal set of states into a set of orthogonal states with a finite probability of success. The optimum strategy is the one that maximizes the average probability of success for this procedure.

However, all the investigations mentioned above concentrate on the local QSD. One would like to know whether the QSD can be remotely implemented in a completely different way compared with a classical state discrimination. To be specific, let us consider two spatially separated parties Alice and Bob. The state to be determined is possessed by Bob, but Alice performs a nonlocal POVM and informs Bob if the outcome is successful, in which case Bob can perform an orthogonal measurement to determine his state. We called this QSD scheme a remote one. Remote QSD, which is a special case of remote interaction, is a critical step for implementation of networked quantum communication processing and distributed quantum computation. It can also be used, e.g., as an efficient remote attack in quantum cryptography.

In this paper, we try to deal with the problem of remote UD. We should extend the scheme introduced in [9] to remote UD. Beside section 1 written as an introduction, the paper is organized as follows. In section 2, we present an explicit scheme for remote unambiguous discrimination of linear independent symmetric d-level quantum states, where the remote generalized measurement described by a positive operator valued measure lies at the heart. We construct the required remote POVM. We also construct a quantum network for realizing the remote POVM with the aid of two-level EPR pairs and LOCCs. In section 3, we summarize our main results and conclusions.

2. Scheme for remote unambiguous discrimination of linear independent symmetric quantum states

Assume that Alice and Bob are involved in the process. Alice possesses a device which is able to perform local POVMs. Further assume that a d-level quantum system (qudit) is prepared in one of the d nonorthogonal states $\{|\psi_l\rangle\}$ lying in a d-dimensional Hilbert space. We restrict ourselves to the case of nonorthogonal linearly independent states, which are symmetric, defined by

$$|\psi_l\rangle = Z^l|\psi_0\rangle \qquad (l = 0, 1, \ldots, d-1), \tag{2}$$

where $|\psi_0\rangle = \sum_{k=0}^{d-1} c_k|k\rangle$ and all real coefficients c_k are nonzero satisfying $\sum_{k=0}^{d-1} c_k^2 = 1$. The action of the Z operator on this state is such that $Z|j\rangle = \exp(i2\pi j/d)|j\rangle$ and $Z^d = I$. We hand Bob the qudit, and inform only Alice what are the possible states $|\psi_l\rangle_B$. We wish to design a remote QSD scheme, which should satisfy the requirement that Bob knows which state his qudit was prepared in, and furthermore, which never gives errors. The trivial realization of remote UD can be finished by the bidirectional quantum state teleportation. The discriminated state is teleported from Bob to Alice. Then Alice performs a local POVM on the state and sends the resulting state back to Bob via another state teleportation. The total resources for this trivial protocol are two edits (entanglement qudits) and four cdits (classical dits) [17]. And there is no restriction for the discriminated states. In general however, the implementation of remote UD with quantum state teleportation methods may not be efficient, and provides only an upper bound on the required amount of entanglement. Since entanglement is an expensive resource, it is important to optimize its usage and search for economic methods for implementing remote UD. For the remote unambiguous discrimination of the quantum states $|\psi_l\rangle_B$, a better way is through the direct teleportation of the quantum operation U_{BA} of equation (1). We have known that the minimum communication cost for teleportation of a certain kind of d-level non-local condition unitary evolution, such as a non-local generalized controlled-NOT gate, is one edit and two cdits [18].

2.1. Nonlocal system-ancilla conditional evolution

Our immediate task is to build up the nonlocal condition unitary evolution U_{BA} of equation (1), which could projects remotely the state $|\psi_l\rangle$ of equation (2) onto a set of orthogonal states $\{|u_l\rangle\}$ and onto another set of linearly dependent sates $\{|\phi_l\rangle\}$. That is, we are only interested in discriminating between a conclusive result and an inconclusive result.

To do this, a two-level auxiliary particle A with the initial state $|\phi\rangle_A = |0\rangle_A$ is introduced by Alice, and a unitary transformation U_{BA}, in the entangled ancilla-system space, based on the basis $\{|00\rangle_{BA}, |10\rangle_{BA}, \ldots, |(d-1)0\rangle_{BA}, |01\rangle_{BA}, |11\rangle_{BA}, \ldots, |(d-1)1\rangle_{BA}\}$ is constructed

$$U_{BA} = \begin{pmatrix} M_0 & M_1 \\ M_1 & -M_0 \end{pmatrix}, \tag{3}$$

where M_0, M_1 are $d \times d$ Kraus operators, and may be expressed as

$$M_0 = \mathrm{diag}(a_0, a_1, \ldots, a_k, \ldots, a_{d-1}), \tag{4}$$

$$M_1 = \mathrm{diag}\left(\sqrt{1-a_0^2}, \sqrt{1-a_1^2}, \ldots, \sqrt{1-a_k^2}, \ldots, \sqrt{1-a_{d-1}^2}\right). \tag{5}$$

Here $|a_k| \leqslant 1 (k = 0, 1, 2, \ldots, d-1)$ can be determined by dividing $c_{\min}$ ($c_{\min} = \min\{|c_k|, k = 0, 1, \ldots, d-1\}$) by the coefficient of the kth term shown in equation (2), for instance, $a_3 = c_{\min}/c_3$.

After applying the nonlocal conditional evolution U_{BA} on the compound system A and B, we get

$$|\psi\rangle_e = U_{BA}|\psi_l\rangle_B|0\rangle_A = M_0|\psi_l\rangle_B|0\rangle_A + M_1|\psi_l\rangle_B|1\rangle_A. \tag{6}$$

From equations (4)–(6), it follows that the nonlocal unitary evolution U_{BA} together with the Alice's conventional local orthogonal measurement is equivalent to the required remote POVM. After Alice measuring her ancilla, we find whether M_0 or M_1 have been generated remotely. M_0 acts to rotate remotely the state $|\psi_l\rangle_B$ to

$$\begin{aligned} M_0|\psi_l\rangle_B &= c_{\min}\{|0\rangle_B + \exp(\mathrm{i}2\pi l/d)|1\rangle_B \\ &\quad + \cdots + \exp[\mathrm{i}2\pi l(j-1)/d]|j-1\rangle_B + \exp(\mathrm{i}2\pi lj/d)|j\rangle_B \\ &\quad + \exp[\mathrm{i}2\pi l(j+1)/d]|j+1\rangle_B + \cdots + \exp[\mathrm{i}2\pi l(d-1)/d]|d-1\rangle_B\} \\ &= (c_{\min}\sqrt{d})\left[(1/\sqrt{d})\sum_{j=0}^{d+1}\exp(\mathrm{i}2\pi lj/d)|j\rangle_B\right] \\ &= (c_{\min}\sqrt{d})|u_l\rangle_B, \end{aligned} \tag{7}$$

in which case Bob can distinguish with certainty among $|\psi_l\rangle_B$ $(l = 0, 1, \ldots, d-1)$ by performing a X_d-MB [19] on his qudit B locally. If the ancilla is observed to be in the state $|1\rangle_A$, M_1 maps the state $|\psi_l\rangle_B$ into

$$\begin{aligned} M_1|\psi_l\rangle_B &= \sqrt{c_0^2 - c_{\min}^2}|0\rangle_B + \sqrt{c_1^2 - c_{\min}^2}\exp(\mathrm{i}2\pi l/d)|1\rangle_B \\ &\quad + \cdots + \sqrt{c_{j-1}^2 - c_{\min}^2}\exp[\mathrm{i}2\pi l(j-1)/d)|j-1\rangle_B \\ &\quad + \sqrt{c_{j+1}^2 - c_{\min}^2}\exp[\mathrm{i}2\pi l(j+1)/d)|j+1\rangle_B \\ &\quad + \cdots + \sqrt{c_{d-1}^2 - c_{\min}^2}\exp[\mathrm{i}2\pi l(d-1)/d)|d-1\rangle_B, \end{aligned} \tag{8}$$

and the inconclusive result is obtained. The success probability of remote UD among $|\psi_l\rangle_B$ $(l = 0, 1, \ldots, d-1)$ is

$$P = \langle\psi_l|_B M_0^+ M_0|\psi_l\rangle_B = d|c_{\min}|^2, \tag{9}$$

which is identical to that in an ordinary local POVM [9].

Some remarks should be made 1. The auxiliary particle is not necessarily a two-level system. However, it has been proved that we cannot extract more quantum information with more probe qubits [20]. That is, the maximal probability is independent of the resources used. Therefore, we can just as well use a two-level qubit as we probe 2. The initial state of the auxiliary qubit can be arbitrary and does not affect the result. We take the initial state as $|0\rangle_A$ for convenience 3. The U_{BA} operator is not unique, there are other similar forms [21].

We have shown above that the remote UD of d linearly independent symmetric quantum states can be realized by performing a nonlocal $2d \times 2d$ conditional evolution U_{BA} on two spatially separated systems (B, A) and a local orthogonal measurement on the ancilla A. For implementation of the nonlocal quantum operation in higher dimensional Hilbert space, one should note that the analog of singlet EPR pair (multi-level EPR pair) including the counterpart in [18] is necessary. However, a multi-level EPR pair is very difficult to be prepared. In contrast, the preparation of the two-level EPR pair can be realized by different schemes [22, 23]. A more important question is that, since a state of qudit can be teleported with the aid of two-level EPR pairs [17], whether the similar version is suitable for teleportation of quantum operation U_{BA}.

2.2. Decomposing the nonlocal system-ancilla conditional evolution into a circuit of two-level controlled gate

It deserves mentioning that the quantum logic network is essential for practical realization of the remote UD in experiment. In the following, we should construct a quantum logic network to realize the nonlocal quantum operation U_{BA} with a set of two-level nonlocal controlled-rotation gates. And a two-level nonlocal controlled-rotation gate can be implemented by using a two-level EPR pair in addition to LOCCs.

There are two usual methods to construct a quantum network. One is to decompose the unitary operation into arbitrary rotation gates on a single qubit and controlled-NOT (CNOT) gates on two qubits [16]. Recently, based on the theory of majorization, Gu *et al* showed the second method to construct the quantum network in realizing deterministic entanglement concentration [24]. In their method, the quantum network is obtained by decomposing the evolution process from the initial state to the final state. Here we construct the needed quantum network using Gu *et al*'s method.

For the unitary evolution defined by equation (6), the initial state reads

$$\begin{aligned}|\psi\rangle_0 &= |\psi_l\rangle_B|0\rangle_A\\ &= c_0|00\rangle_{BA} + c_1\exp(\mathrm{i}2\pi l/d)|10\rangle_{BA} + \cdots + c_{j-1}\exp[\mathrm{i}2\pi l(j-1)/d]|(j-1)0\rangle_{BA}\\ &\quad + c_j\exp(\mathrm{i}2\pi lj/d)|j0\rangle_{BA} + c_{j+1}\exp[\mathrm{i}2\pi l(j+1)/d]|(j+1)0\rangle_{BA}\\ &\quad + \cdots + c_{d-1}\exp[\mathrm{i}2\pi l(d-1)/d]|(d-1)0\rangle_{BA},\end{aligned} \tag{10}$$

and the final state reads

$$\begin{aligned}|\psi\rangle_e = c_{\min}\{&|00\rangle_{BA} + \exp(\mathrm{i}2\pi l/d)|10\rangle_{BA} + \cdots + \exp[\mathrm{i}2\pi l(j-1)/d]|(j-1)0\rangle_{BA}\\ &+ \exp(\mathrm{i}2\pi lj/d)|j0\rangle_{BA} + \exp[\mathrm{i}2\pi l(j+1)/d]|(j+1)0\rangle_{BA}\\ &+ \cdots + \exp[\mathrm{i}2\pi l(d-1)/d]|(d-1)0\rangle_{BA}\}\\ &+ \sqrt{c_0^2 - c_{\min}^2}|01\rangle_{BA} + \sqrt{c_1^2 - c_{\min}^2}\exp(\mathrm{i}2\pi l/d)|11\rangle_{BA}\\ &+ \cdots + \sqrt{c_{j-1}^2 - c_{\min}^2}\exp[\mathrm{i}2\pi l(j-1)/d)|(j-1)1\rangle_{BA}\\ &+ \sqrt{c_{j+1}^2 - c_{\min}^2}\exp[\mathrm{i}2\pi l(j+1)/d)|(j+1)1\rangle_{BA}\\ &+ \cdots + \sqrt{c_{d-1}^2 - c_{\min}^2}\exp[\mathrm{i}2\pi l(d-1)/d]|(d-1)1\rangle_{BA}.\end{aligned} \tag{11}$$

According to equations (6), (10) and (11), the components of $|\psi\rangle_0$ evolve in the following manner:

$$c_0|00\rangle_{BA} \Rightarrow c_{\min}|00\rangle_{BA} + \sqrt{c_0^2 - c_{\min}^2}|01\rangle_{BA}, \tag{12a}$$

$$c_1|10\rangle_{BA} \Rightarrow c_{\min}|10\rangle_{BA} + \sqrt{c_1^2 - c_{\min}^2}|11\rangle_{BA}, \tag{12b}$$

$$\ldots$$

$$c_{j-1}|(j-1)0\rangle_{BA} \Rightarrow c_{\min}|(j-1)0\rangle_{BA} + \sqrt{c_{j-1}^2 - c_{\min}^2}|(j-1)1\rangle_{BA}, \tag{12c}$$

$$c_j|j0\rangle_{BA} \Rightarrow c_{\min}|j0\rangle_{BA}, \tag{12d}$$

$$c_{j+1}|(j+1)0\rangle_{BA} \Rightarrow c_{\min}|(j+1)0\rangle_{BA} + \sqrt{c_{j+1}^2 - c_{\min}^2}|(j+1)1\rangle_{BA}, \tag{12e}$$

$$\ldots$$

$$c_{d-1}|(d-1)0\rangle_{BA} \Rightarrow c_{\min}|(d-1)0\rangle_{BA} + \sqrt{c_{d-1}^2 - c_{\min}^2}|(d-1)1\rangle_{BA}. \quad (12f)$$

Correspondingly, we decompose the evolution process from $|\psi\rangle_0$ to $|\psi\rangle_e$ into d steps, where each step implements one of the evolutions (12). This can be realized by applying a unitary operation to qubit A under the control of different logical states of system B. The evolutions (12a)–(12f) correspond respectively to the control conditions of qudit B which are in the logical states $|0\rangle, |1\rangle, \ldots, |j-1\rangle, |j\rangle, |j+1\rangle, \ldots,$ and $|d-1\rangle$. In the first step, the evolution (12a) is realized by applying to qubit A a unitary operation U_0 satisfying

$$U_0|0\rangle_A = (c_{\min}/c_0)|0\rangle_A + \sqrt{1-(c_{\min}/c_0)^2}|1\rangle_A, \quad (13)$$

if and only if B is in $|0\rangle$. In the basis $\{|0\rangle, |1\rangle\}$, the matrix of U_0 can be written as

$$\begin{aligned} U_0 &= \begin{pmatrix} c_{\min}/c_0 & -\sqrt{1-(c_{\min}/c_0)^2} \\ \sqrt{1-(c_{\min}/c_0)^2} & c_{\min}/c_0 \end{pmatrix} \\ &= \begin{pmatrix} \cos(\theta_0/2) & -\sin(\theta_0/2) \\ \sin(\theta_0/2) & \cos(\theta_0/2) \end{pmatrix} \equiv U_y(\theta_0), \end{aligned} \quad (14)$$

where $U_y(\theta_0)$ is the rotation operator about the y-axis [16]. We can see that the evolution of equation (12a) is just a nonlocal two-level controlled-rotation transformation

$$U_{B^0A} = |0\rangle_B\langle 0|_B \otimes [U_y(\theta_0)]_A + |1\rangle_B\langle 1|_B \otimes I_A. \quad (15a)$$

In the same way, the evolutions (12b)–(12f) are realized respectively by applying the nonlocal controlled-rotation transformations

$$U_{B^1A} = |1\rangle_B\langle 1|_B \otimes [U_y(\theta_1)]_A + |0\rangle_B\langle 0|_B \otimes I_A, \quad (15b)$$

$$\ldots$$

$$U_{B^{j-1}A} = |j-1\rangle_B\langle j-1|_B \otimes [U_y(\theta_{j-1})]_A + |0\rangle_B\langle 0|_B \otimes I_A, \quad (15c)$$

$$U_{B^jA} = I, \quad (15d)$$

$$U_{B^{j+1}A} = |j+1\rangle_B\langle j+1|_B \otimes [U_y(\theta_{j+1})]_A + |0\rangle_B\langle 0|_B \otimes I_A, \quad (15e)$$

$$\ldots,$$

$$U_{B^{d-1}A} = |d-1\rangle_B\langle d-1|_B \otimes [U_y(\theta_{d-1})]_A + |0\rangle_B\langle 0|_B \otimes I_A \quad (15f)$$

on qubit A and on the respective logical state $|1\rangle, \ldots, |j-1\rangle, |j\rangle, |j+1\rangle, \ldots, |d-1\rangle$ of qudit B, where

$$\begin{aligned} U_y(\theta_i) &= \begin{pmatrix} c_{\min}/c_i & -\sqrt{1-(c_{\min}/c_i)^2} \\ \sqrt{1-(c_{\min}/c_i)^2} & c_{\min}/c_i \end{pmatrix} \\ &= \begin{pmatrix} \cos(\theta_i/2) & -\sin(\theta_i/2) \\ \sin(\theta_i/2) & \cos(\theta_i/2) \end{pmatrix}. \end{aligned} \quad (16)$$

Summarizing, U_{BA} can be decomposed into the product of $(d-1)$ two-level nonlocal controlled-rotation transformations

$$U_{BA} = \sum_{k=0}^{d-1} U_{B^kA} \qquad (k \neq j), \quad (17)$$

where U_{B^kA} acts non-trivially only on two vector components of the d-dimensional Hilbert space H_B and two-dimensional Hilbert space H_A. Since $[U_{B^kA}, U_{B^{k\prime}A}] = 0$ for any $k, k\prime = 0, 1, \ldots, j-1, j+1, \ldots, d-1$, we need not distinguish their order. By equation (17) we simplify the problem in operation of U_{BA}, for the quantum operation which acts on a two-level system is far more feasible than that on a multi-level system.

2.3. Implementing nonlocal controlled-rotation gate using a two-level EPR pair

Now our task is to construct another quantum network for implementing the nonlocal controlled-rotation gate of equations (15), for example,

$$U_{B^kA} = |k\rangle_B\langle k|_B \otimes [U_y(\theta_k)]_A + |0\rangle_B\langle 0|_B \otimes I_A \qquad (k = 1, 2, \ldots, j-1, j+1, \ldots, d-1), \tag{18}$$

by using a two-level EPR pair $|E\rangle_{ab}^k = (|00\rangle + |kk\rangle)_{ab}$ and LOCCs. (Here and in what follows, we leave out normalization factors for states.). We give qubit a to Alice and qubit b to Bob. The following operations must be performed

Firstly, Alice performs a local controlled-rotation gate

$$U_{aA} = |0\rangle_a\langle 0|_a \otimes [U_y(\theta_k)]_A + |k\rangle_a\langle k|_a \otimes I_A \tag{19}$$

on her qubits a and A followed by a local Hadamard gate H_a^k ($|0\rangle \to |0\rangle + |k\rangle$, $|k\rangle \to |0\rangle - |k\rangle$) on qubit a.

Then, Alice measures the state of qubit a and sends the result to Bob. When the result is $|k\rangle_a$, Bob needs to perform a σ_Z^k gate ($|0\rangle \to |0\rangle$, $|k\rangle \to -|k\rangle$) on qubit b, but no operation is applied otherwise. After that, we have

$$c_k|k\rangle_B|0\rangle_A(|00\rangle + |kk\rangle)_{ab} \quad \Rightarrow \quad c_k|k\rangle_B\{|k\rangle_b|0\rangle_A + |0\rangle_b[U_y(\theta_k)|0\rangle_A]\}. \tag{20}$$

Lastly, Bob measures the state of his qubit b, and the measurement result is communicated to Alice. As soon as Alice is informed of Bob's result, she can perform an appropriate unitary transformation (according to the two possible results $|k\rangle_b$ and $|0\rangle_b$, the corresponding transformation are $U_y(\theta_k)$ and I) on qubit A to obtain the state

$$\begin{aligned} c_k|k\rangle_B\{|k\rangle_b|0\rangle_A + |0\rangle_b[U_y(\theta_k)|0\rangle_A]\} &\Rightarrow c_k|k\rangle_B[U_y(\theta_k)|0\rangle_A] \\ &= U_{B^kA}(c_k|k\rangle_B|0\rangle_A). \end{aligned} \tag{21}$$

This completes the implementation of nonlocal unitary operation U_{B^kA}. The implementation consumes one two-level EPR pair represented by the state $|E\rangle_{ab}^k$, and one cbit in each direction to communicate the measurement outcomes.

To summarize, to complete the transformation of equation (6) the present scheme requires $(d-1)$ two-level EPR pairs and $2(d-1)$ cbits in total. The distinct advantage of this scheme is that only a multichannel which is made up of $(d-1)$ two-level EPR pairs is used. And a two-level EPR pair is easy, being contrary to the multi-level EPR pair, to be prepared.

3. Summary

We have proposed a scheme for remote unambiguous discrimination of a set of equally likely, symmetric, linearly independent, d-dimensional nonorthogonal states. Our scheme has been designed for obtaining the optimal value of local conclusive measurements, which is given by Chefles bound [9]. The scheme consists of a remote POVM. We explicitly construct the required remote POVM, which can be realized by performing a nonlocal $2d \times 2d$ unitary operation on Alice's ancilla A and Bob's qudit B, and the conventional local orthogonal measurement on ancilla. The implementation of the remote POVM in this scheme demands a small-scale quantum communication network. By decomposing the evolution process from the initial state to the final state, we construct a quantum network for realizing the remote POVM with the aid of two-level EPR pairs and LOCCs, and thus provide a physical means to realize the remote UD. This method is convenient for a large class of nonlocal quantum processes involving remote POVM. The interest in the remote UD is not 'academic', remote UD can be used, e.g., as an efficient remote attack in quantum cryptography.

Recently, an experimental scheme for local unambiguous discrimination of four linearly independent symmetric states based on linear optics only have been proposed [25]. A nonlocal two-level CNOT gate and a nonlocal arbitrary single qubit rotation gate have been implemented experimentally in a linear optics setup as reported in [26, 27]. Therefore, we believe the scheme in our paper will be realized in experiment.

Acknowledgment

This work is supported by the Natural Science Foundation of Guangdong Province, China (Grant No. 06029431).

References

[1] Bennett C H 1992 *Phys. Rev. Lett.* **68** 3121
[2] Ekert A 1992 *Phys. Rev. Lett.* **67** 661
[3] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[4] Chefles A 1998 *Phys. Lett.* A **239** 339
[5] Ivanovic I D 1987 *Phys. Lett.* A **123** 257
[6] Dieks D 1988 *Phys. Lett.* A **126** 303
[7] Peres A 1988 *Phys. Lett.* A **128** 19
[8] Jaeger G and Shimony A 1995 *Phys. Lett.* A **197** 83
[9] Chefles A and Barnett S M 1998 *Phys. Lett.* A **250** 223
[10] Bergou J, Hillery M and Sun Y 2000 *J. Mod. Opt.* **47** 487
[11] Roa L, Retamal J C and Saavdra C 2003 *Phys. Rev.* A **66** 012103
[12] Gopiath T, Das R and Kumar A 2005 *Phys. Rev.* A **71** 042307
[13] Huttner B, Muller A, Gautier J D, Zbinden H and Gisin N 1996 *Phys. Rev.* A **54** 3783
[14] Clarke R, Chefles A, Barnett S and Riis E 2001 *Phys. Rev.* A **63** 040305R
[15] Mohseni M, Steinberg A and Bergou J 2004 *Phys. Rev. Lett.* **93** 200403
[16] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[17] Zhou J D, Hou G and Zhang Y D 2001 *Phys. Rev.* A **64** 012301
[18] Chen L B, Lu H and Jin R B 2007 *Chin. Phys.* **16** 3204
[19] Deng F G, Li X H, Li C Y, Zhou P and Zhou H Y 2006 arXiv:quant-ph/0605214
[20] Hsu L Y 2002 *Phys. Rev.* A **66** 12308
[21] Bin H and Bergou J 2006 *Phys. Lett.* A **356** 306
[22] Hagley E, Maitre X, Nogues G, Wunderlich C, Brune M, Raimoud J M and Haroche S 1997 *Phys. Rev. Lett.* **79** 1
[23] Tuchette Q A, Wood C S, King B E, Myatt C J, Leibfried D, Itano W M, Monroe C and Wineland D J 1998 *Phys. Rev. Lett.* **819** 3631
[24] Gu Y J, Li W D and Guo G C 2006 *Phys. Rev.* A **73** 022321
[25] Jiménez O, Sánchez X, Delgado A and Saavedra C 2007 arXiv:quant-ph/07053813
[26] Huang Y F, Ren X F, Zheng Y S, Duan L M and Guo G C 2004 *Phys. Rev. Lett.* **93** 240501
[27] Xiang G Y, Li J and Guo G C 2005 *Phys. Rev.* A **71** 044304